Quiz 15 Solution

April 20, 2018

1. (2 points) Find the exact displacement (do not round) of a particle from $t = 2$ to $t = 3$ if the velocity function is $v(t) = \frac{t+3}{t}$.

   Solution: The displacement from $t = 2$ to $t = 3$ is given by $s(3) - s(2)$. By FTC,

   $$\begin{aligned} s(3) - s(2) &= \int_2^3 s'(t)\,dt \\ &= \int_2^3 \frac{t+3}{t}\,dt \text{ since } v(t) = s'(t) \\ &= \int_2^3 \frac{t}{t} + \frac{3}{t}\,dt \\ &= \int_2^3 1 + 3 \cdot \frac{1}{t}\,dt \\ &= (t + 3\ln|t|)\Big|_2^3 \\ &= (3 + 3\ln 3) - (2 + 3\ln 2) \\ &= 1 + 3\ln(3) - 3\ln(2) \\ &= 1 + \ln(27/8) \text{ by log rules} \end{aligned}$$

   Answer: $1 + \ln(27/8)$

2. (1 point) Use the Trapezoidal Rule to approximate $\int_1^3 2\sqrt{x}\,dx$ with $n = 4$. Round to 4 decimal places.

   Solution: We know $T_4 = \frac{1}{2}\Delta x\big(f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)\big)$.
   Since the interval is $[1, 3]$ and $n = 4$, $\Delta x = \frac{3-1}{4} = \frac{1}{2}$.
   So $T_4 = \frac{1}{2} \cdot \frac{1}{2}\big(2\sqrt{1} + 2 \cdot 2\sqrt{1.5} + 2 \cdot 2\sqrt{2} + 2 \cdot 2\sqrt{2.5} + 2\sqrt{3}\big) \approx 5.5861$

   Answer: 5.5861

3. (1 point) Evaluate $\int_1^3 2\sqrt{x}\,dx$. Round your answer to 4 decimal places.

   Solution: We know $\int 2x^{1/2}\,dx = \frac{4}{3}x^{3/2} + C$. By FTC,

   $$\begin{aligned} \int_1^3 2\sqrt{x}\,dx &= \frac{4}{3}x^{3/2}\Big|_1^3 \\ &= \frac{4}{3}(3^{3/2} - 1^{3/2}) \\ &\approx 5.5949 \end{aligned}$$

   Answer: 5.5949

4. (1 point) What do you most need to review for the final exam?

   Answer: Answers will vary.